

, Member, IEEE,

, Senior Member, IEEE

Abstract—

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Index Terms—



1 INTRODUCTION

The problem of t -wise δ -diversity [1] is to find a set S of n elements from a domain \mathcal{U} such that any subset T of S of size t has δ distinct elements. This is a generalization of the well-known problem of t -wise δ -diversity [1]. In this paper, we study the problem of t -wise δ -diversity with respect to a set of m sets $\mathcal{C} = \{C_1, C_2, \dots, C_m\}$ of elements from \mathcal{U} . We assume that each C_i has size $|C_i| = c_i$ and \mathcal{C} is t -wise δ -diverse. We consider the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} . We study the complexity of this problem and show that it is NP-hard. We also study the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} and S is t -wise δ -diverse. We show that this problem is also NP-hard. We study the complexity of this problem and show that it is NP-hard. We also study the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} and S is t -wise δ -diverse. We show that this problem is also NP-hard.

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The problem of t -wise δ -diversity [1] is to find a set S of n elements from a domain \mathcal{U} such that any subset T of S of size t has δ distinct elements. This is a generalization of the well-known problem of t -wise δ -diversity [1]. In this paper, we study the problem of t -wise δ -diversity with respect to a set of m sets $\mathcal{C} = \{C_1, C_2, \dots, C_m\}$ of elements from \mathcal{U} . We assume that each C_i has size $|C_i| = c_i$ and \mathcal{C} is t -wise δ -diverse. We consider the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} . We study the complexity of this problem and show that it is NP-hard. We also study the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} and S is t -wise δ -diverse. We show that this problem is also NP-hard. We study the complexity of this problem and show that it is NP-hard. We also study the problem of finding a set S of n elements from \mathcal{U} such that S is t -wise δ -diverse with respect to \mathcal{C} and S is t -wise δ -diverse. We show that this problem is also NP-hard.

2 "FOLLOWING" LINK CASCADE MODEL

ing links

neighbor-

$$G = (V, E, t),$$

$$v \in V$$

$$e_{uv} \in E$$

$$u, v.$$

B,

$$t : E \rightarrow \cup \{\perp\}$$

$$t(e_{uv}) = n \in$$

$$e_{uv}$$

$$n,$$

()

-

)

-

C,

A

1. Diffusion effect between links decays over time.

bility $g_{e'e}$

discovery proba-

A

f

f

f

f

f

f

f

f

f

f

f

f

f

f

f

f

f

"following" link cascade model.

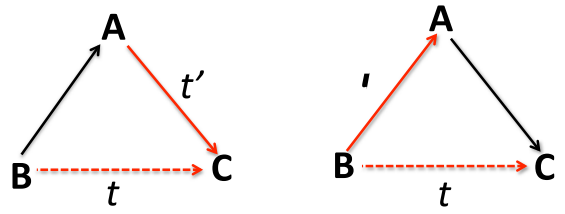
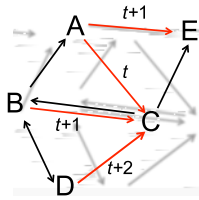
Organization.

$$t' + \lambda$$

$$\delta$$

$$e.$$

$$f$$



(a) Follower diffusion

2. (. . . , $\delta = 7$) ,

e, e'

(A, B, C)

$t' - 1$

$t' - 1, B$

A, C

B

Followee diffusion.

$t, t' \leq t \leq t' + \delta,$

$(t' - 1),$

e_{BA}

e_{BC}

followee diffusion.

e_{AC}

$t + 1$

e_{BC}

e_{DC}

$t + 2$

A

e_{AE}

$t + 1$

$()$

Follower diffusion.

$t, t' \leq t \leq t' + \delta,$

$t' - 1,$

e_{AC}

e_{BC}

follower diffusion.

3 DATA AND OBSERVATIONS

e_{AC}

e_{CB}

ff

3.1 Data Collection

0,000

0/ / 0 0 / / 0 0.

0/ / 0 0 / / 0 0.

()

3.2 Observations

13 24

1 12

()

()

t'

t

$0 \leq t - t' \leq \delta$

C_{Δ}

B_{Δ} C_{Δ}

$[t', t' + \delta]$ $|C_{\Delta}^+|$

Δ r_{Δ}

-
-

Pattern significance.

$$r_{\Delta} = \frac{|C_{\Delta}^+|}{|C_{\Delta}|} \quad (1)$$

$$\begin{pmatrix} + & - \\ A & C \\ C & B \end{pmatrix} C(+ - 0)$$

4 MODEL LEARNING

Likelihood function.



$$\theta = \{h_{e'e}, g_{e'e}\}$$

(e', e) .

$$\begin{pmatrix} g_{e'e} & h_{e'e} \\ (e', e) \end{pmatrix} \begin{pmatrix} 0.0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$y_{e'e} = \prod_{t=t'}^{t_e} (1 - g_{\Delta})^{t-t'} + (1 - h_{\Delta}) \prod_{t=t'}^{t_e} (1 - g_{\Delta})^{t-t'-1}$$

$$y_{e'e} = 1 - h_{\Delta} g_{\Delta} \sum_{t=t'}^{t_e} (1 - g_{\Delta})^{t-t'} \quad (6)$$

$$= h_{\Delta} (1 - g_{\Delta})^{t_e - t' + 1} + (1 - h_{\Delta})$$

$$\log \mathcal{L} = \sum_{e \in \mathcal{E}} \left\{ \log \sum_{\vec{\alpha}_{S_e}} \prod_{e' \in S_e} x_{e'e}^{\alpha_{e'}} y_{e'e}^{1 - \alpha_{e'}} + \sum_{e' \in R_e} \log y_{e'e} \right\}$$

EM algorithm.

$$q(e|\vec{\alpha}_{S_e}) = \frac{p(e|\vec{\alpha}_{S_e})}{\sum_{e' \in S_e} p(e'|\vec{\alpha}_{S_e})}$$

$$\log \mathcal{L} = \sum_{e \in \mathcal{E}} \left\{ \log \sum_{\vec{\alpha}_{S_e}} \hat{q}(e|\vec{\alpha}_{S_e}) \frac{p(e|\vec{\alpha}_{S_e})}{\hat{q}(e|\vec{\alpha}_{S_e})} + \sum_{e' \in R_e} \log y_{e'e} \right\}$$

$$\geq \sum_{e \in \mathcal{E}} \left\{ \sum_{\vec{\alpha}_{S_e}} \hat{q}(e|\vec{\alpha}_{S_e}) \log \frac{p(e|\vec{\alpha}_{S_e})}{\hat{q}(e|\vec{\alpha}_{S_e})} + \sum_{e' \in R_e} \log y_{e'e} \right\},$$

$$\hat{q}(e|\vec{\alpha}_{S_e}) \log \hat{q}(e|\vec{\alpha}_{S_e})$$

$$Q(\theta, \hat{\theta})$$

$$Q(\theta, \hat{\theta}) = \sum_{e \in \mathcal{E}} \left\{ \sum_{\vec{\alpha}_S} \right.$$

$$h_{\Delta} = \frac{\sum_{(e',e) \in C_{\Delta}^+} \hat{D}_{e'e} + \sum_{(e',e) \in C_{\Delta}^-} \hat{B}_{e'e}}{|C_{\Delta}|}, \quad (12)$$

$$g_{\Delta} = \frac{\sum_{(e',e) \in C_{\Delta}^+} \hat{A}_{e'e}}{\sum_{(e',e) \in C_{\Delta}^-} \hat{B}_{e'e}(\delta + 1) + \sum_{(e',e) \in C_{\Delta}^+} \hat{D}_{e'e}(t_e - t_{e'} + 1)}. \quad (13)$$

$$D_{e'e} = B_{e'e} + A_{e'e} - A_{e'e}B_{e'e}. \quad (14)$$

| A | | 1. | |
|---------------------------------------|--------------|--------------|--------------|
| $G = (V, E, t)$ | | | |
| $\theta = \{h_{\Delta}, g_{\Delta}\}$ | | | |
| $(0,)$ | | | |
| E- | $e \in S$ | $x_{e'e}$ | $y_{e'e}$ |
| | $e' \in S$ | $A_{e'e}$ | $B_{e'e}$ |
| 0 | $e' \in R$ | $D_{e'e}$ | $B_{e'e}$ |
| | $\Delta = 1$ | h_{Δ} | g_{Δ} |
| Convergence | | | |

5 APPLICATIONS

Followee maximization.

$i = 1$ to $|H|$

$u \in V \setminus S$

$s_u = 0$

$r = 1$ to $|H|$

$s_{u+} = |FCM(S \cup \{u\})|$

$s_u = s_{u+}/R$

$S = S \cup \{argmax_{u \in V \setminus S} s_u\}$

Followee maximization.

$i = 1$ to $|H|$

$u \in V \setminus S$

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$s_u = s_{u+}/R$

$S = S \cup \{argmax_{u \in V \setminus S} s_u\}$

| A | | 2. | | / | |
|---|--|----|--|---|--|
| $G = (V, E), v, k$ | | | | | |
| $S = \emptyset, R = 0,000$ | | | | | |
| $i = 1$ to $ H $ | | | | | |
| $u \in V \setminus S$ | | | | | |
| $s_u = 0$ | | | | | |
| $r = 1$ to $ H $ | | | | | |
| $s_{u+} = FCM(S \cup \{u\}) $ | | | | | |
| $s_u = s_{u+}/R$ | | | | | |
| $S = S \cup \{argmax_{u \in V \setminus S} s_u\}$ | | | | | |

6 EXPERIMENTS

6.1 Experimental Setup

Experimental Setup

$i = 1$ to $|H|$

$u \in V \setminus S$

$s_u = 0$

$r = 1$ to $|H|$

$s_{u+} = |FCM(S \cup \{u\})|$

$s_u = s_{u+}/R$

$S = S \cup \{argmax_{u \in V \setminus S} s_u\}$

Evaluation metrics.

$$p(e|S_e)$$

$$p(e|S_e) > \tau$$

$$p(e|S_e) > \tau$$

$$p(e|S_e) > \tau$$

$$p(e|S_e) > \tau$$

$$p(e|S_e) > \tau$$

$$p(e|S_e) > \tau$$

Comparison methods.

Basic.

$$SVM$$

LRC

LRC

Collaborative filtering (CF):

$$CF_score(u, v) = \sum_w I(w, v) sim(w, u),$$

$$sim(w, u)$$

$$I(w, v)$$

$$CF_score(u, v).$$

SimRank.

$$0$$

Katz.

Random-random model (RR).

$$RR_score(u, v) = \frac{1}{|F(u)|} \sum_w I(u, w) I(w, v) \frac{1}{|F(w)|},$$

$$|F(u)|$$

$$RR_score(u, v).$$

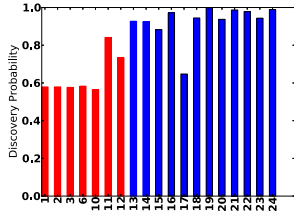
Preferential attachment with communities (PAC).

$$PAC_score(u, v) = \beta \left(\alpha \frac{|N(v)|}{\sum_{v \in C(u)} |N(v)|} + (1 - \alpha) \frac{1}{|C(u)|} \right) + (1 - \beta) \left(\alpha \frac{|N(v)|}{\sum_{v \in V} |N(v)|} + (1 - \alpha) \frac{1}{|V|} \right),$$

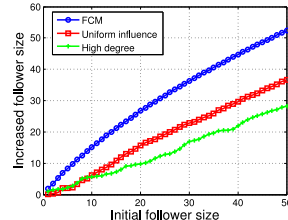
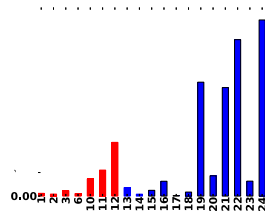
$$|N(v)|$$

$v, C(u)$

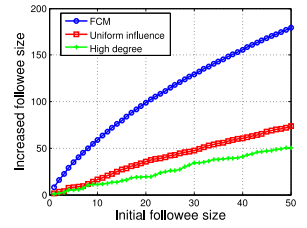
f $u,$ $0.V$ f ff
 f $u,$ f f
 f $u.$ $f\alpha$ βf 0 $0.$ $f-$
 $-$



(a) Discovery Probabilities



(a) Follower maximization



(b) Followee maximization

Per-triad analysis.

Delay analysis.

Convergence analysis.

Model parameter analysis.

6.3 Application Improvement

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ff
ff ff ff ff
ff f /f
ff

7 RELATED WORK

Diffusion model and influence maximization.

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ff

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